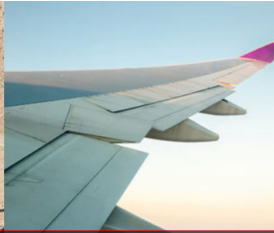




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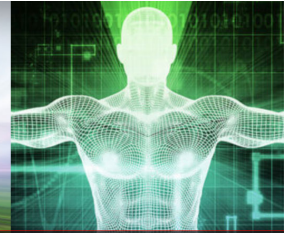
AEROSPACE



DEFENSE



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MEDICAL



2021 High Torque Skew Axis Gearing

A TECHNICAL PRIMER

F. EVERTZ, M. GANGIREDDY, B. MORK, T. PORTER & A. QUIST

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Introduction

All types of gearing can be classified by the spatial relationship of the axis of one member relative to the axis of the other member. The axes relationship can be parallel, intersecting, or skewed. Of these, the latter is probably the least understood of all gearing types. There are three major types of skew axis gearing: worm gears, Hypoid®, and Spiroid® gears. Worm gears have been used since gears were called “cogs” and the technology has been well chronicled ever since. Hypoid gears were developed many years ago by Gleason and are well understood and widely used in the automotive industry. The third major type of skew axis gears, Spiroid gears, are the subject of this paper. For reference, the various types of skew axis gears are depicted in *Figure 1* below.

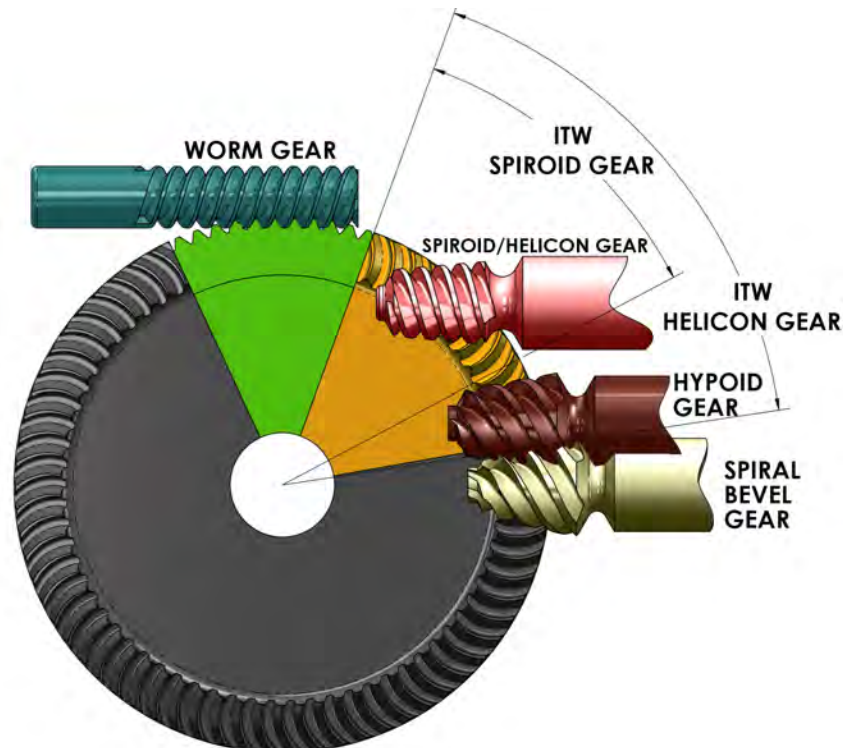


Figure 1

Spiroid Gearset Characteristics

Spiroid® gear sets comprise the family of gearing defined by the following physical characteristics:

- ✓ Each gearset is comprised of a pinion and a gear.
- ✓ The pinion is a threaded worm type member comprised of one or more threads which mesh with the gear which is a “face” type gear.
- ✓ Each tooth on the gear is spiral in shape, thus there is a concave side and a convex side of each tooth. The concave side has a higher pressure angle than the convex side. Thus there is a “hi side” pressure angle and a “lo side” pressure angle.
- ✓ The pinion and gear axes are skewed and are spaced a specific distance apart, called the “center distance.” The two axes are typically perpendicular but can vary by as much as ± 20 degrees from perpendicular.
- ✓ The gearset is considered either “right hand” or “left hand” by convention. The “hand” is determined by the orientation of the pinion in spatial relation to the gear. This relationship also determines the rotational direction of each member relative to the other.
- ✓ The threaded pinion is conical in shape with a typical taper angle of 5 degrees (10 deg. included angle) but the taper angle can vary by 5 to 10 deg. The face of the gear is also conical in shape having a typical cone angle of 8 deg. which can also vary by a few deg.
- ✓ The gear ratios can range from about 10:1 to as high as 400:1

See *Figure 2* for a typical Spiroid® gearset arrangement.

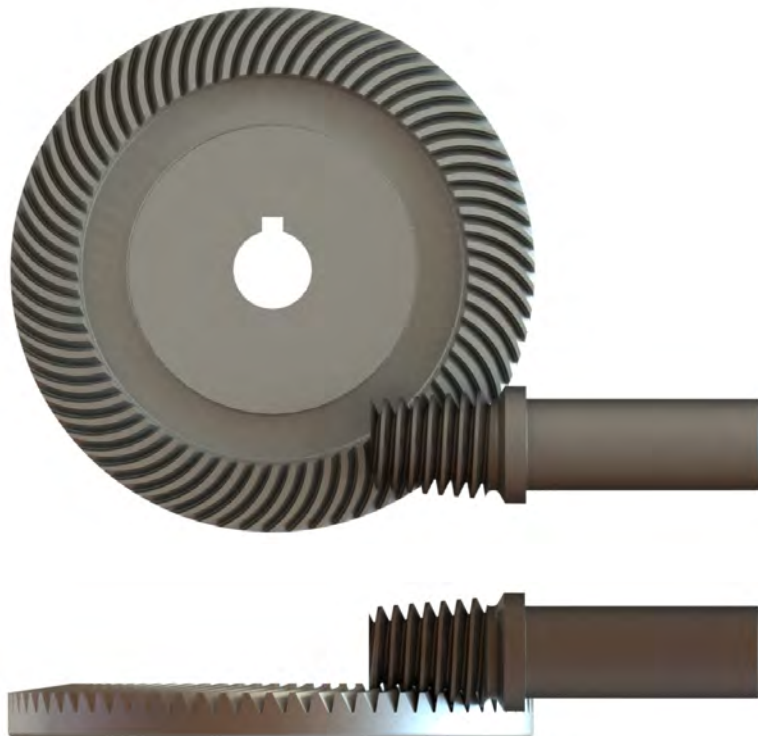


Figure 2

A very widely used subset of Spiroid gears is Helicon® gearing. Helicon gears are very similar to Spiroid gears, but the distinguishing feature is that the pinion is cylindrical in shape rather than conical. The gear face, likewise, is flat rather than tapered. Thus the gear face angle and pinion taper angle, τ , are zero. See *Figure 3*. Helicon gearset ratios can be as low as approximately 4:1.

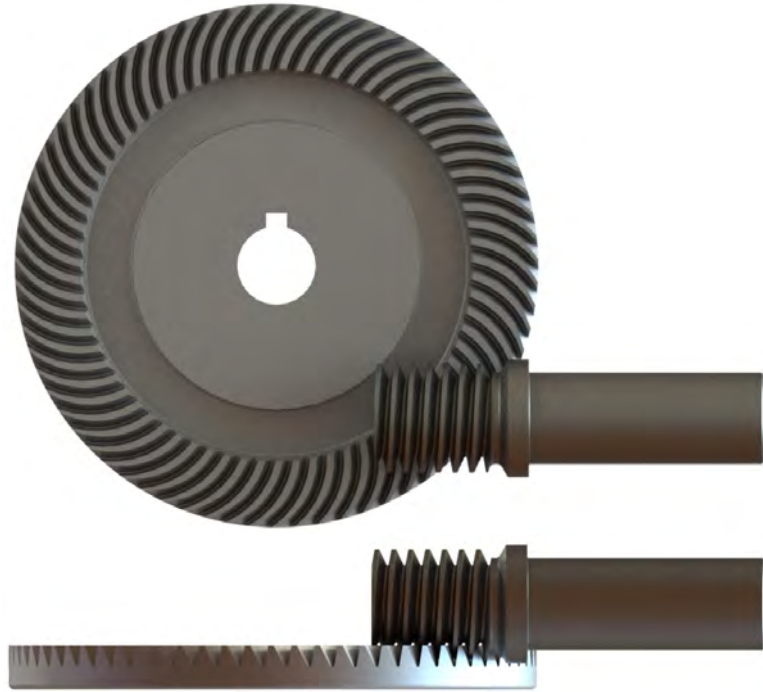


Figure 3

Basic Theory

The pinion is the defining member of a Spiroid gearset, i.e., the pinion thread design in turn determines the gear tooth design. For every pinion thread system there exists a primary pitch cone which defines an ideal tooth curve to satisfy the requirements for conjugate action with a mating gear. (Figure 4)

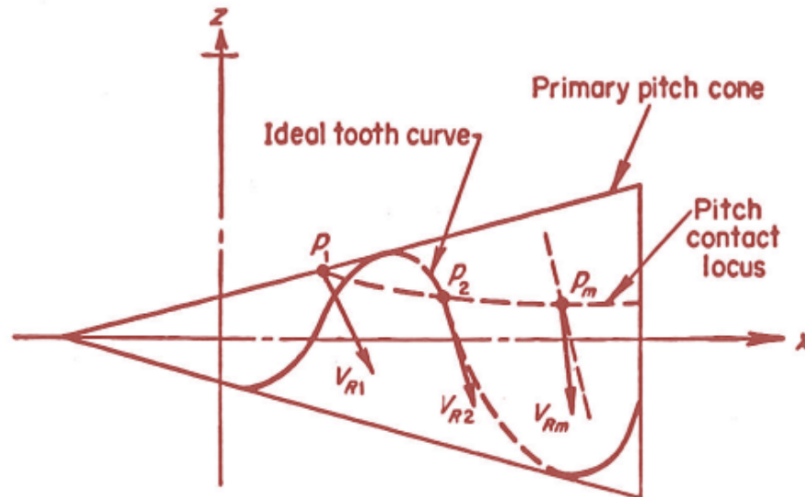


Figure 4

Any particular point on the ideal tooth curve can be chosen as the “pitch point” for the gearset. (Figure 5) It is this pitch point which determines all other tooth contact parameters.

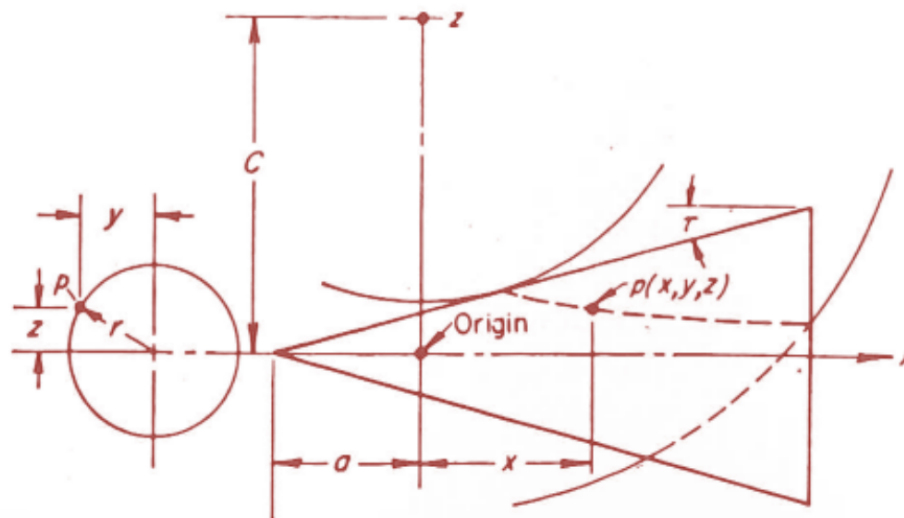


Figure 5

The location of the pitch point is somewhat arbitrary, but experience has shown it is best suited to lie on the outside cone (or in the case of Helicon gears, on the outside cylinder) of the pinion threads and about one third of the distance across the gear face as shown in *Figure 6*.

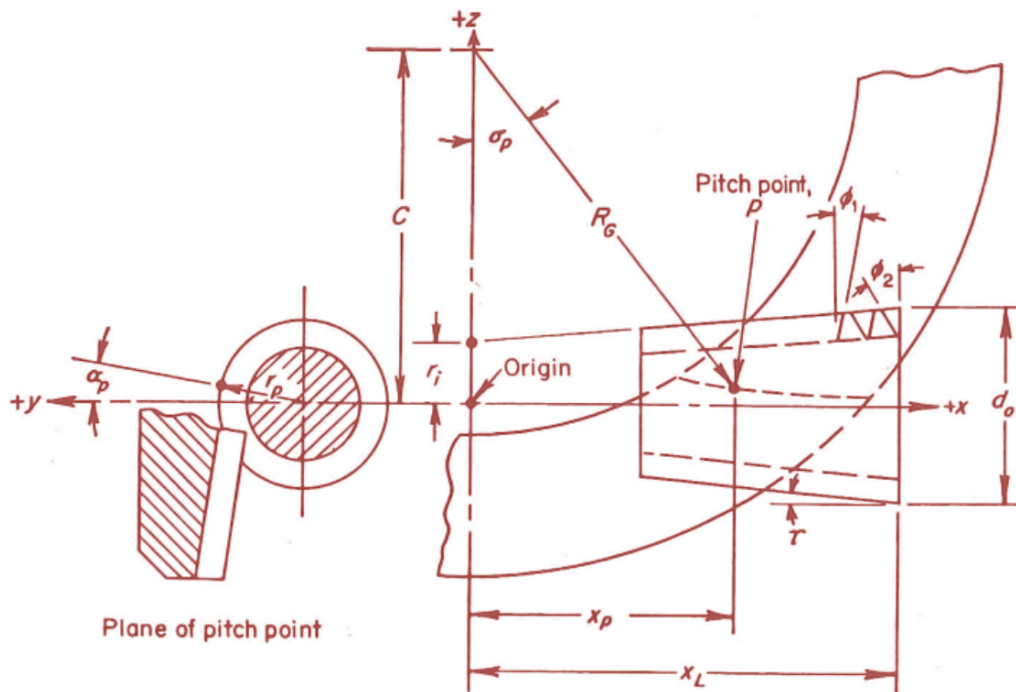


Figure 6

From the chosen pitch point location and knowing the values of the pinion taper angle and center distance, the r , z , and y values are:

$$r = (a + x) \tan \tau \quad (1)$$

$$z = \frac{C}{1 + \frac{x}{r \tan \tau}} \quad (2)$$

$$y = \sqrt{r^2 - z^2} \quad (3)$$

And with the gearset ratio, N/n , known the lead, L , of the ideal tooth curve at the pitch point is:

$$L = \frac{2\pi(C-z)}{\frac{N}{n} \frac{xy}{r^2}} \quad (4)$$

It is important to keep in mind that this value is the *conical* lead, not the cylindrical lead. See Figure 7.

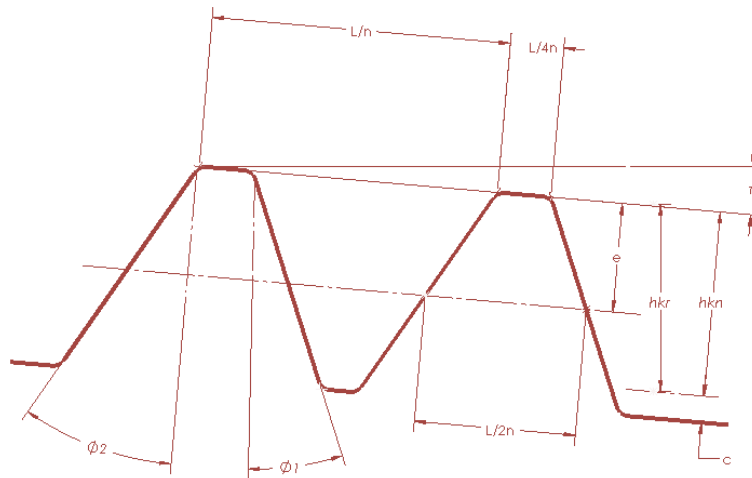


Figure 7

In the case of Helicon gears, r is the outside radius of the pinion, $z=0$, and $y=r$.

Referring to Figure 5 (above) and Figure 8 (below), the Pitch point coordinates and Lead can be expressed in terms of angle parameters α and σ .

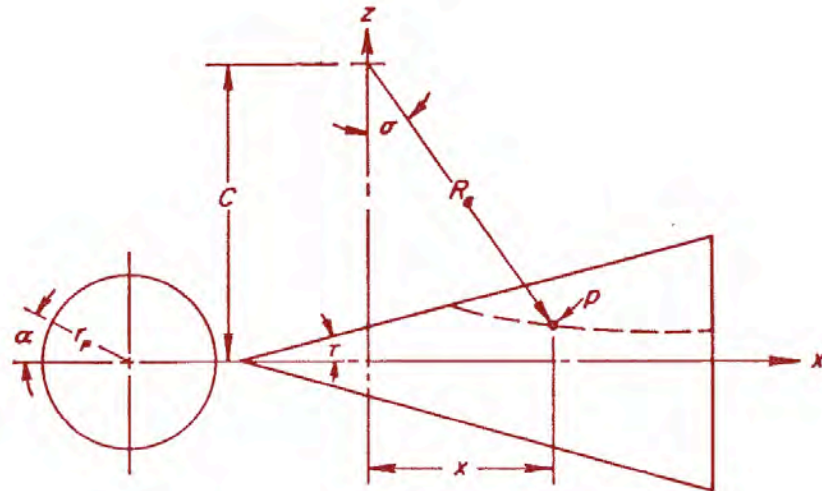


Figure 8

$$x = R_G \sin \sigma \quad (5)$$

$$y = r_p \cos \alpha \quad (6)$$

$$z = r_p \sin \alpha \quad (7)$$

When Equations 5, 6, and 7 are substituted into Equation 4:

$$L = \frac{2\pi R_G \cos \sigma}{\frac{N}{n} - \left(\frac{R_G}{r_p}\right) \sin \sigma \cos \alpha} \quad (8)$$

Tooth and blank proportions

Given the pinion lead and pitch point coordinates, the pinion threads and gear blank proportions can be defined. Good design practice combined with many years of experience has led to guidelines for determining optimum pinion threads and subsequent gear tooth and blank parameters. These guidelines have come to be known as “standard proportions.” It should be kept in mind that special situations can necessitate deviating from these guidelines, but that should only be done with a great deal of caution and expertise.

Standard pitch point angle

Referring to *Figure 6*, the pitch point angle, σ , can range from 30 to 60 deg., but for general purpose design, 40 deg. works best and is considered the standard pitch point angle. The center distance, C , is directly related to the pitch point angle. A 40 deg. pitch point angle results in a center distance of one third of the gear outside dia.

Pinion taper angle

Although the pinion taper angle, τ , can range from zero to 10 deg., five degrees has been considered the standard taper angle for Spiroid pinions (zero for Helicon pinions).

Due to the offset center distance, C , the gear face angle of eight deg. most closely corresponds to the 5 deg. pinion taper angle. Therefore 8 deg. is considered the standard Spiroid gear face angle.

From *Figure 6*, the pinion radius where the projected cone crosses the gear centerline is:

$$r_i = r_p - R_G \sin \sigma_p \tan \tau \quad (9)$$

This is known as the “zero plane radius.”

And the pinion diameter at the large end is:

$$d_o = 2(r_i + x_L \tan \tau) \quad (10)$$

Other Spiroid pinion and gear blank proportions that result from $\sigma = 40$ and $\tau = 5$ are:

$$R_o = 1.5C$$

$$R_i = 1.1C$$

$$X_L = 1.2C$$

$$F = 0.75C$$

Likewise, Helicon gear and pinion blank proportions are:

$$R_o = 1.75C$$

$$R_i = 1.1C$$

$$X_L = 1.5C$$

$$F = 0.75C$$

It should be pointed out that for Helicon gearset ratios below 10:1, center distances must decrease significantly for proper operation and highly specialized analysis is required.

Pinion thread proportions

Referring to *Figure 7*, good design practice has shown that the thread tip thickness should be approximately half the tooth thickness, or $L/4n$.

With this established as a “standard” proportion, the pinion addendum, working depth, and full depth of thread can be established. Assuming the axial thread thickness at mid working depth is $L/2n$, gives

$$\text{addendum, } e = \frac{\frac{L}{4n}}{(\tan(\phi_2 - \tau) + \tan(\phi_1 + \tau))} \quad (11)$$

$$h_{kn} = 2e \quad (12)$$

Further, it has been established that the minimum clearance, c , should be:

$$c = 0.07 \left(\frac{L}{n}\right) + 0.002 \quad (13)$$

And pinion thread root radius should be, $r_r = 0.75c$

Pinion spiral angle

Another important geometric feature is the pinion spiral angle, often called the “thread angle.” The mean thread angle is defined as the angle between a line tangent to the thread and a line normal to the pinion axis, both lines passing through a point at mid thread working depth. This angle is used, among other things, to calculate gearset efficiencies. For purposes of calculating the mean thread angle, we must define a mean x dimension, x_m . For standard proportions, x_m has been defined as $x_m = 0.83 C$.

From this we can get the mean thread angle pinion radius, r_m .

$$r_m = r_i - \frac{h_{kr}}{2} + x_m \tan \tau \quad (14)$$

From which the mean thread angle, ϕ , can be found:

$$\tan \phi_m = \frac{L \sec \tau}{2\pi r_m} \quad (15)$$

Gearset Sizing

Sliding velocity

All skew axis gears are characterized by predominantly sliding action in the gear mesh.

Sliding velocity is defined as the vector difference in instantaneous velocities of the gear and pinion at any point of contact. The calculation of the value of instantaneous sliding velocity is very complex mathematically and is beyond the scope of this paper. However, it is a very key parameter in designing Spiroid gearing since it has a direct influence on both gearset durability and efficiency. Therefore, it has been convenient to develop a graphical approach to determining sliding velocity.

Figure 9 is a plot of gear ratio vs. the “sliding velocity factor,” V_R , for various center distances. V_R is essentially the sliding velocity for a pinion speed of 1 RPM. Thus the sliding velocity, V_S , is:

$$V_S = V_R \times \text{pinion speed (RPM)} \quad (16)$$

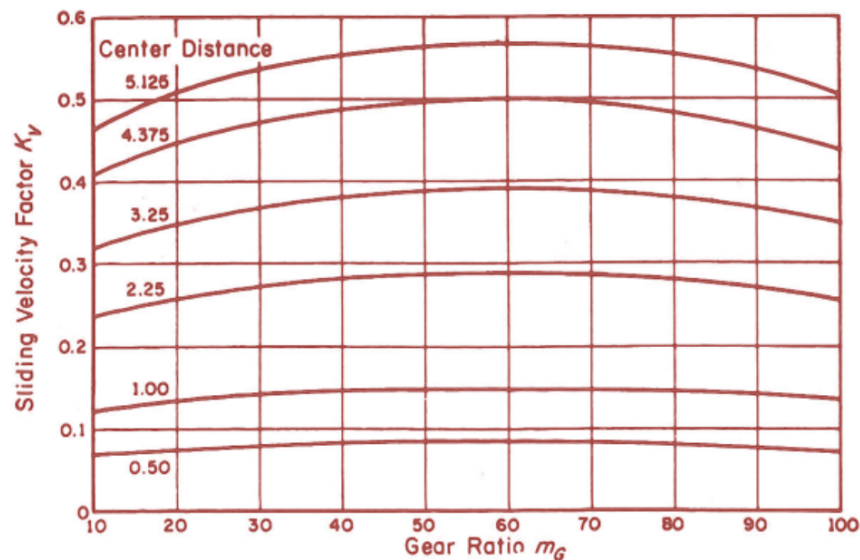


Figure 9

Gearset failures

Spiroid gearset failure modes can be grouped into three categories: pinion thread bending failure, gear tooth shear failure, and tooth surface durability. Of these failure modes, surface durability is by far the most critical since thread bending and gear tooth shear failures would happen at much higher loads.

Durability

Field and laboratory testing as well as over sixty years of application experience has resulted in the method of defining the gearset (durability) rating. This is defined as the power level at which surface failure will not occur. This rating of course is based on properly mounted and lubricated gearsets.

In terms of horsepower,

$$P_o = \frac{T_{GO}\omega_G}{63025} \quad (17)$$

$$T_{GO} = K_w \rho_o l_c R_e \quad (18)$$

These factors are based on the mean contact point which is at gear mid-face and pinion mid-tooth height. See *Figure 10*.

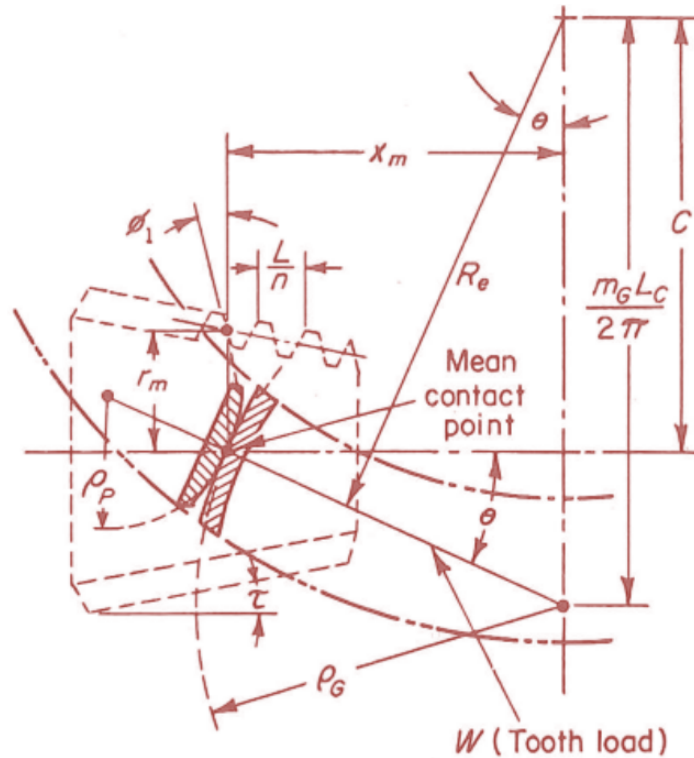


Figure 10

K_w is the “allowable wear factor” and has been developed through testing and field experience and is a function of sliding velocity.

Figure 11 illustrates K_w for two of the most commonly used combinations of materials and lubricants.

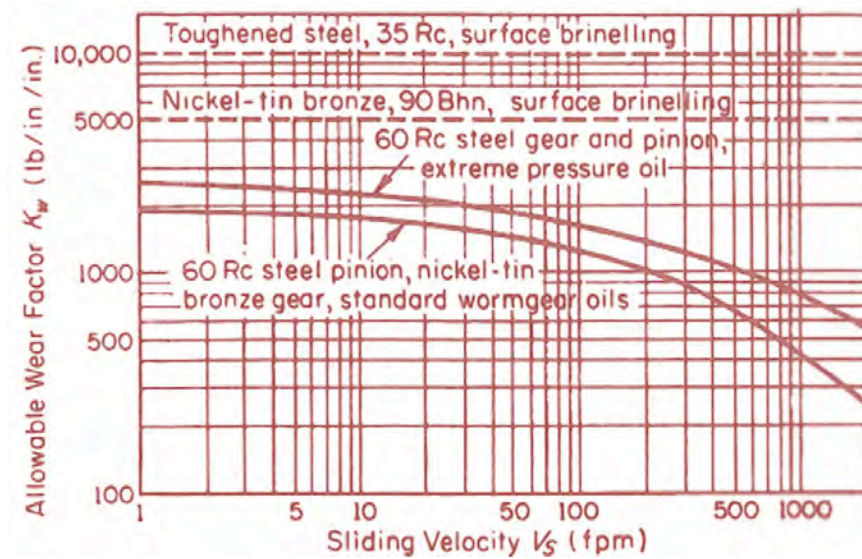


Figure 11

ρ_o is the *relative* radius of curvature and is a critical parameter in determining the contact stress between two curved surfaces and, by extension, gearset torque capacity. To determine ρ_o , we need to determine the equivalent *cylindrical* lead from:

$$L_{c1} = L (1 - \tan \tau \tan \phi_1) \quad (19)$$

Angle θ can be found from:

$$\tan \theta = \frac{\frac{m_G L_{c1}}{2\pi} - C}{x_m} \quad (20)$$

The radii of curvature of the gear tooth and pinion thread are:

$$\rho_G = x_m \sec \theta \quad (21)$$

$$\rho_P = \frac{r_m}{\tan \phi_1 \cos^3 \theta} \quad (22)$$

and the *relative* radius of curvature is:

$$\frac{1}{\rho_o} = \frac{1}{\rho_G} + \frac{1}{\rho_P} \quad (23)$$

l_c is the total length of the effective load carrying contact lines. The length of a line of contact between any pinion thread and mating gear tooth is basically equal to the working depth. Thus the total length is the working depth times the number of teeth in simultaneous contact, or "contact ratio," C_R .

Referring to *Figure 12*, the value of dimension A can be found from:

$$A = \sqrt{R_o^2 - C^2} - \sqrt{R_i^2 - C^2} \quad (24)$$

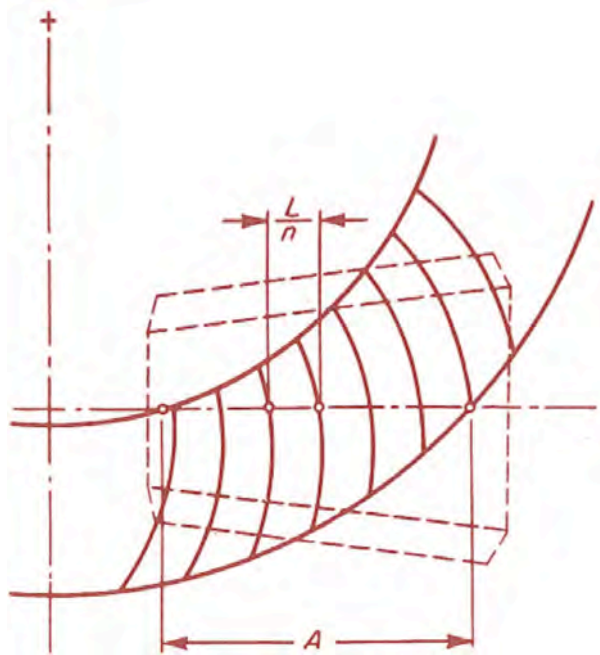


Figure 12

The contact ratio can then be calculated from:

$$C_R = \frac{A}{\frac{L}{n}} \quad (25)$$

And the total length of contact line is:

$$l_c = C_R h_{kr} \quad (26)$$

Effective gear radius, R_e , as shown in *Figure 10*, is determined from:

$$R_e = \frac{m_G L_{c1}}{2\pi} \cos \theta \quad (27)$$

Gearset torque capacity can now be calculated from equation (18)

$$T_{GO} = K_w \rho_o l_c R_e \quad (18)$$

Efficiency

Just as with worm gears or any other gear types with sliding mesh characteristics, efficiency is an important consideration in designing Spiroid or Helicon gears. Generally speaking, the higher the gear ratio, the lower the expected efficiency. Gearset efficiency is not, however, a direct function of gearset ratio. The parameters which directly affect efficiency are pressure angles, spiral angles, and friction coefficient.

When the pinion is the driving member, the gearset efficiency, E_p , is:

$$E_p = \frac{\cos \phi_n + \mu \cot \phi_G}{\cos \phi_n + \mu \cot \phi_P} \quad (28)$$

Conversely, when the gear is the driving member (gearset is “backdriving”), the gearset efficiency, E_G , is:

$$E_G = \frac{\cos \phi_n - \mu \cot \phi_P}{\cos \phi_n - \mu \cot \phi_G} \quad (29)$$

It is apparent (and intuitive) that the gear driving efficiency is always significantly lower than when the pinion is the driving member. Actually, for certain geometric combinations, the gear driving efficiency can be zero, or have a negative value. This is known as “self locking,” a condition which has sometimes been used to advantage since it prevents backdriving (the “poor man’s brake”). If this is a design intent, however, it should only be done under well controlled and static conditions.

The friction coefficient, μ , can be a very elusive value since it is a function of several variables, such as surface roughness, materials, and lubrication. However, it has been the subject of intense study through the ages, and much laboratory data has been accumulated. Thus it has been well established that μ varies with sliding velocity.

Figure 13 shows friction coefficient, μ , as a function of sliding velocity.

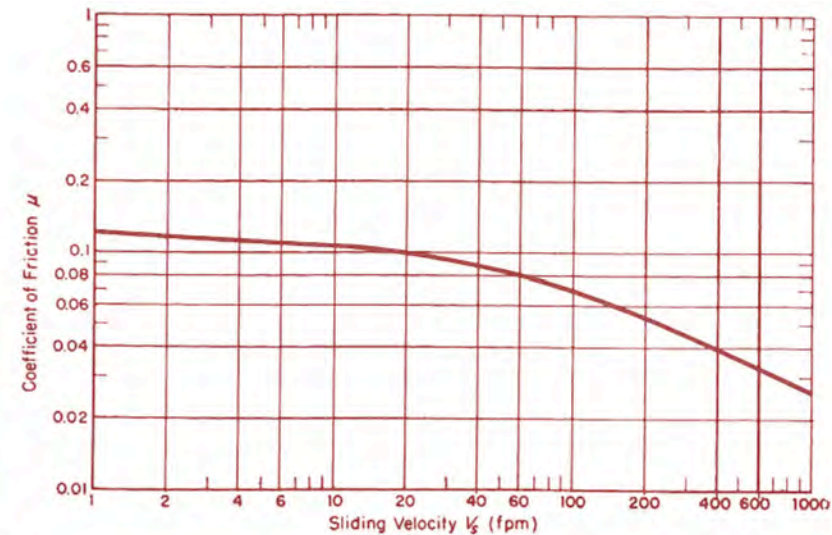


Figure 13

Efficiency calculations must be based on a specific point or location, and it is convenient to use the already established pitch point parameters for this purpose.

The pinion and gear spiral angles can be found from:

$$\tan \phi_P = \frac{L \sec \tau}{2\pi r_P} \quad (30)$$

$$\sin \phi_G = m_G \frac{r_P}{R_G} \sin \phi_P \quad (31)$$

It has also been found convenient to determine the pinion spiral angle for which self-locking occurs.

$$\cot \phi_P = \frac{\cos \phi_n}{\mu} \quad (32)$$

Application Engineering

Tooth load components

In order to facilitate other gear engineering design tasks such as determining bearing loads, shaft stresses and deflections, etc., it has proven useful to break the gear output torque into tooth load components using a standard set of xyz coordinate axes. For any given gearset design, tooth load components corresponding to these axes, f_x , f_y , and f_z , can be determined. Units for these components are lbs. per inch pound of gear output torque. Thus total tooth loads corresponding to the xyz system are F_x , F_y , and F_z , where

$$F_x = f_x \times T_G \quad (33)$$

$$F_y = f_y \times T_G \quad (34)$$

$$F_z = f_z \times T_G \quad (35)$$

Figure 14 illustrates a left hand gearset showing the direction of the tooth load components for both high and low side drives. These load components as shown are acting on the gear member. The load components acting on the pinion are equal in magnitude but in the opposite direction.

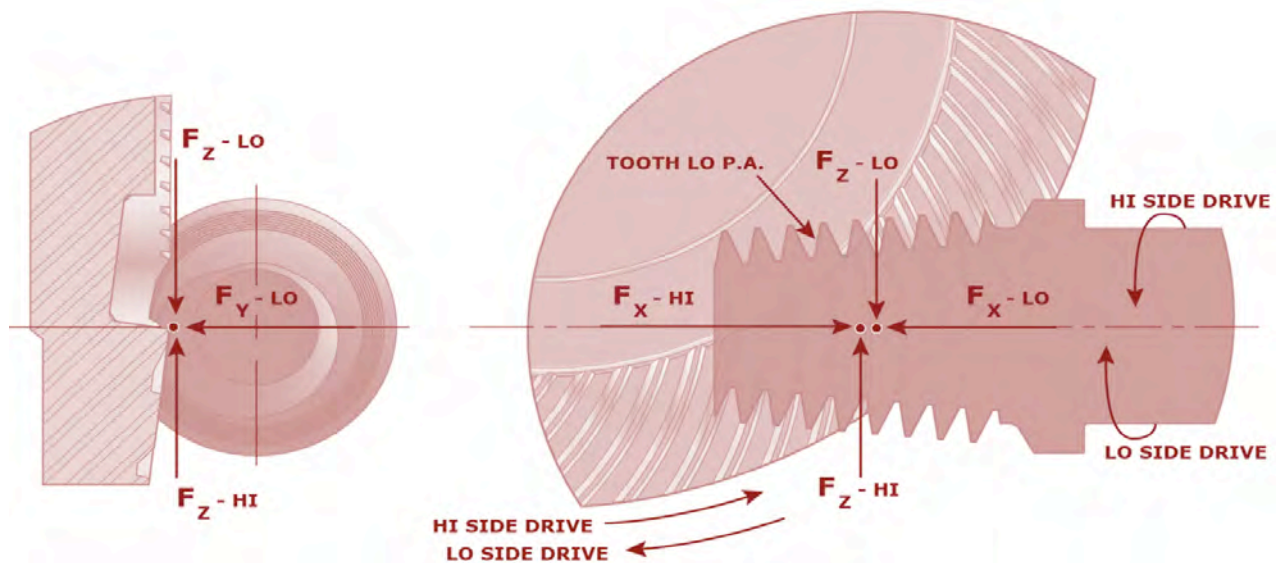


Figure 14

Backlash

As with all types of gearing, backlash must always be taken into account. Backlash can be designed into a gearset and controlled in several ways. It can be accomplished by deliberately thinning the pinion thread or gear tooth by a specific amount during the manufacturing process. The more typical method of controlling backlash is by adjustment of the axial location of gear or pinion with respect to the mating member. In a typical Spiroid gearbox assembly this is usually accomplished by adding or subtracting shims at a location such that the gear moves further in or further out of mesh with the pinion.

To avoid “trial and error” shimming for backlash adjustment, it is convenient to determine the relationship between shim thickness change and backlash change.

When the gear is adjusted, the backlash change, Δb is defined as:

$$\Delta b = \Delta s (\tan \phi_1 + \tan \phi_2) \quad (36)$$

where Δs is the change in shim thickness

When very fine backlash control is required, it is possible to adjust a Spiroid pinion along its axis. In this case relatively large changes in shim thickness can effect very small changes in backlash. This technique, of course, is not possible with Helicon gearsets. When the Spiroid pinion is adjusted, the backlash change is:

$$\Delta b = \Delta s \tan \tau (\tan \phi_1 + \tan \phi_2) \quad (37)$$

Materials

As with most gear types, there is a very wide range of material choices for Spiroid or Helicon gears and pinions. Material selection is dictated by a number of factors including the intended application, gearset size, and operating environment.

In the vast majority of applications, the pinion material choice is steel. For highly loaded, high performance applications it is recommended that pinion threads be carburized and hardened. For lesser demanding applications, through hardening is often sufficient.

There is a broader selection of materials for Spiroid or Helicon gears. As with pinions, steel is the most typical material choice. Carburized and hardened or through hardened steel is a typical choice for gears in the small to medium size range (one to six inches in outside diameter). For gears larger than eight to ten inches in outside diameter, aluminum bronze has proven to be a good choice since heat treat distortion issues are avoided. For gears requiring very high accuracy, bronze is also the material of choice.

For high production applications there are also other options for lower cost such as powdered metal or injection molded plastic gears.

When selecting material for Spiroid or Helicon gears it is important to be aware that the gearset torque rating shown in equation 19 is based on carburized and hardened steel only. For other material combinations the torque rating is reduced. The Table below shows de-rating factors for several material combinations.

Gear material	De-rating factor
Aluminum Bronze	0.80
Powdered Iron Alloy Sintered and hardened	0.65
Aluminum (die cast)	0.35
Molded plastic	0.25

Lubrication

Spiroid and Helicon gears belong to the family of sliding action gears, and as such lubrication is every bit as important as gear detail design or material selection.

As with material selection, there is a wide range of choices for lubricants as well as methods of lubrication, i.e., how the lubricant is delivered to the gear mesh. For Spiroid and Helicon gears there are, however, some very specific guidelines.

Grease lubrication can be considered in many applications but these should be limited to those of lighter duty and lower speed where generated heat is not an issue. Some examples are hand operated drives, lightly loaded instrument gears, and gearsets intended only for motion transmission.

As a lubricant, oil is preferred in situations where loads and speeds are more demanding. Oil lubricants should always be selected from the family of EP (extreme pressure) oils. There are those known as “mild EP” oils, which usually contain lead additives and are suitable for lighter duty or motion transmission gearing. For higher performance applications, oil containing the additives sulfur, chlorine or phosphorus should be specified. These are known as “true EP” oils which, under high temperatures and pressure, form a tenacious film between contacting surfaces which effectively prevents welding or galling between contacting surfaces.

For low to moderate speeds, splash lubrication can be adequate. For low speeds and high loads, it is recommended that the gear mesh is submerged in the oil bath. However, that is not recommended for very high speeds since high churning losses and resultant temperature spikes can result. Instead, an oil jet or spray into mesh system should be considered.

Nomenclature

A = Approximate length of contact
 a = Distance along pinion axis between pitch cone apex and common perpendicular to gear and pinion axis
 C = Center distance
 c = Clearance
 C_R = Contact ratio
 d_o = Pinion outside diameter
 E_G = Efficiency of gear driving
 E_P = Efficiency of pinion driving
 e = Addendum
 F = Face width of pinion
 h_{kn} = Working tooth depth normal to thread tips
 h_{kr} = Working tooth depth normal to pinion axis
 K_v = Sliding velocity factor
 K_w = Allowable wear factor
 L = Conical lead of pinion
 l_c = Total length of contact line
 L_{C1} = Equivalent cylindrical lead on low side
 M = Material factor
 m_G = Gear ratio
 N = Number of teeth on gear
 n = Number of teeth on pinion
 R_G = Pitch radius of gear
 R_i = Point radius of gear
 r = Any pinion radius
 r_i = Intercept of element of pinion pitch cone with center distance
 r_r = Thread root radius
 r_m = Mean plane radius of pinion at midface and half the working depth
 r_0 = Largest radius of pinion pitch cone
 r_p = Pitch radius of pinion
 T_G = Output torque of gear
 T_{GB} = Output torque of gear at breaking load
 T_{GO} = Output torque of gear at allowable tooth load
 V_s = Relative sliding velocity
 W_{GO} = Allowable wear load on tooth of gear
 x_L = Distance along pinion axis between cone section of largest diameter and common perpendicular to gear and pinion axis
 x_m = Co-ordinate to plane of mean radius of pinion
 y_h = Distance along gear axis between tips of teeth at point radius and common perpendicular to gear and pinion axis
 α = Angular co-ordinate of pitch point on pinion
 σ = Pitch-point location angle on gear
 τ = Angle between an element of pinion-pitch cone and pinion axis
 τ_G = Angle between an element of gear-blank face and a perpendicular to the gear axis
 Φ_1 = Tooth axial pressure angle (low side)
 Φ_2 = Tooth axial pressure angle (high side)
 Φ_2' = Critical pressure angle
 ω = Angular velocity
 ω_G = Angular velocity of gear
 ω_P = Angular velocity of pinion
 φ_G = Mean spiral angle
 φ_P = Pinion spiral angle
 θ = Position angle of mean point
 ρ_G = Radius of curvature of gear tooth
 ρ_P = Radius of curvature of pinion thread
 ρ_o = Relative radius of curvature